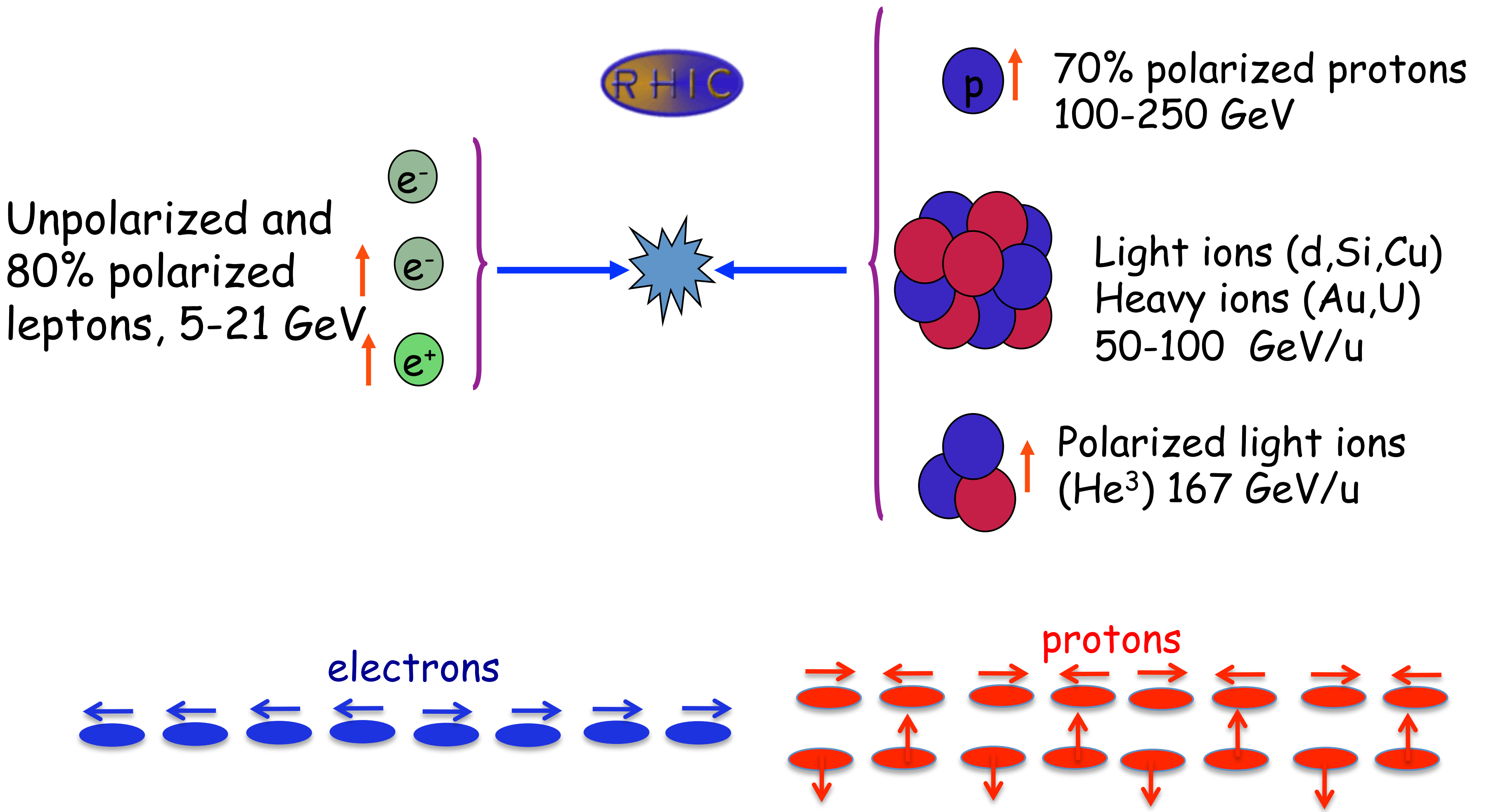
Lattice studies: Orbits control in eRHIC

- Orbits control
  - FFAG as a bent FODO beam-line
  - Matching of arcs to linac, path-length control
  - Merging orbits in regular straight sections, bypass
  - Separating colliding beam
- Multiple parameter optimization & Questions remained

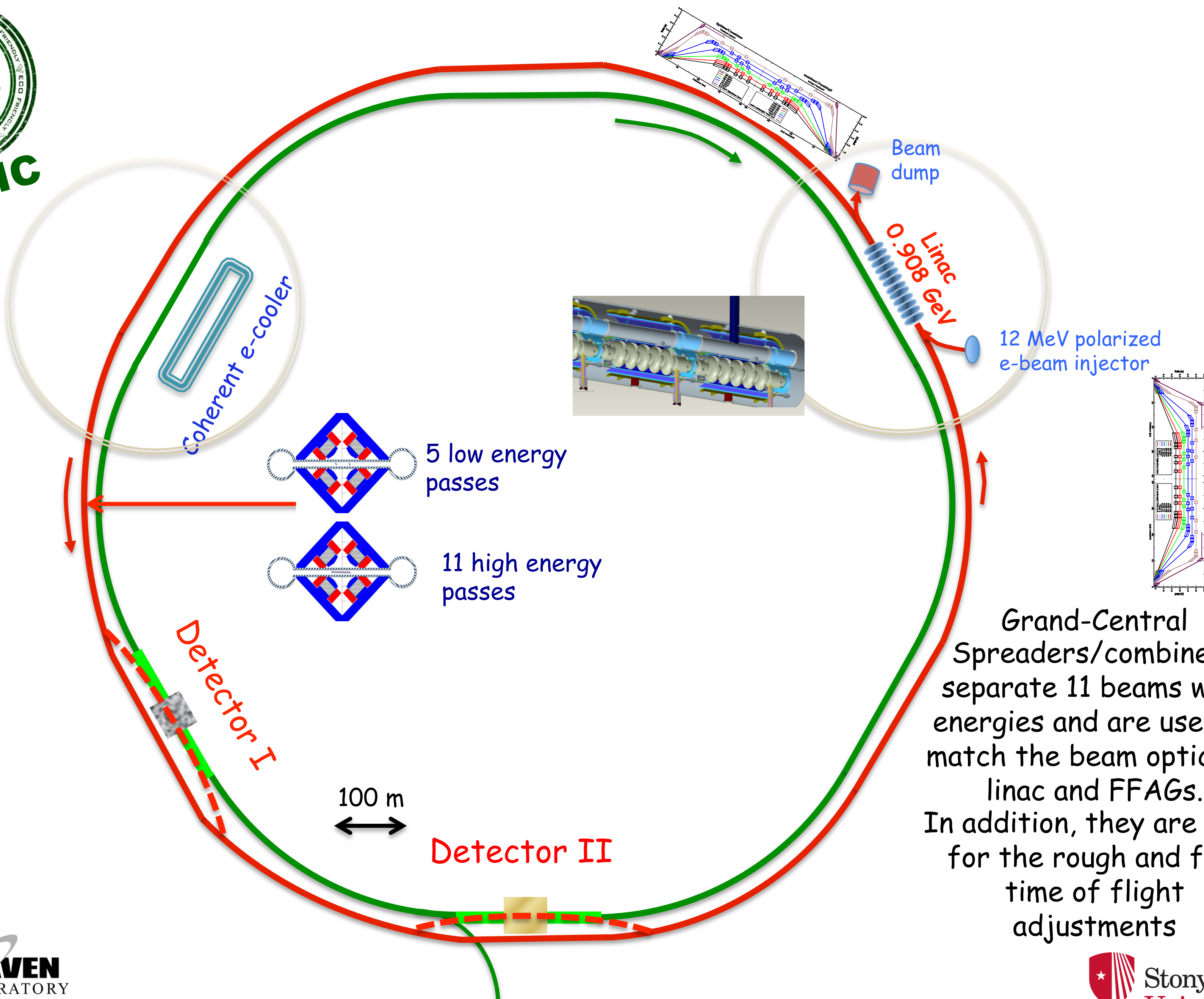
Vladimir N. Litvinenko
Stony Brook University, Stony Brook, NY, USA
Brookhaven National Laboratory, Upton, NY, USA
Center for Accelerator Science and Education

eRHIC: QCD Facility at BNL

Add electron accelerator to the existing $2B RHIC



eRHIC with 10 GeV FFAG ERL



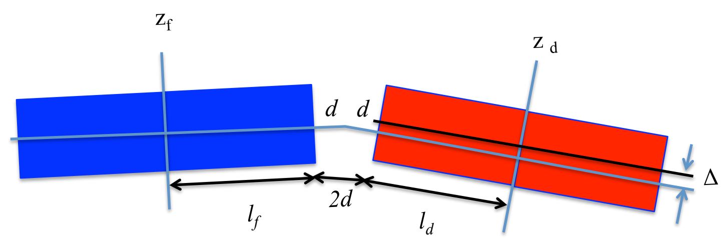
Grand-Central Spreaders/combiners separate 11 beams with energies and are used to match the beam optics of linac and FFAGs.
In addition, they are used for the rough and fine time of flight adjustments

Main accelerator elements of eRHIC concept

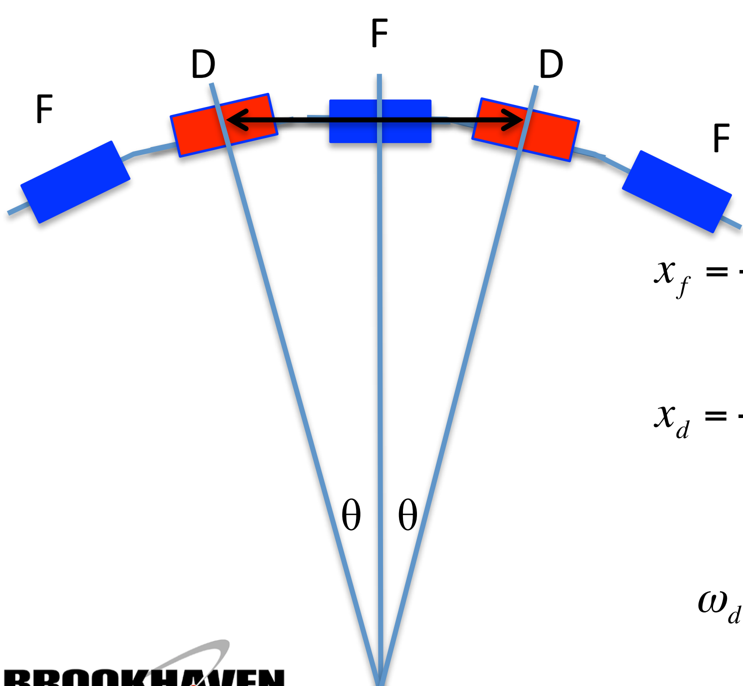
- Use 16-pass ERL for electrons to reach high luminosity at high energy
- Use combination of our previous design (arcs + splitters/combiners) with cost effective FFAG arcs
- Reduce SRF frequency to further improve HOM damping and beam stability – major requirement for increasing number of passes from 6 to 16
- Use 2 FFAG arcs made of shifted quadrupoles to transport 16 (=5+11) beams
- Use splitters and combiners to match optics functions to linac ($\beta_{x,y}$, $\alpha_{x,y}$, $D_{x,y}$, $D'_{x,y}$, R_{56}) and time of flight – 10 parameters for each energy (160 parameter for eRHIC). This can not be done otherwise *(or at least this was not demonstrated)*
- Use harmonic jump to collide electrons with 50 GeV and 250 GeV hadrons
- Use 1.322 GeV linac to have longitudinal polarization in both IRs
- Use a gap to eliminate ion capture and fast ion instability
- Use a BPM bunch within a gap to track all orbits everywhere..

eRHIC FFAG arcs as a bent FODO beam-line

- An ideal eRHIC FFAG cell is comprised of two quadrupoles (F & D) whose magnetic axis are shifted horizontally with respect to each other by Δ
- The structure has a natural bilateral symmetry, e.g. all extrema (min and max) are in the centers of the quadrupoles *(independently of any approximation!)*
- Orbit dependence on the energy can be easily found in paraxial approximation
- **Everything can be done accurately and analytically – no doubt that proposed FFAG lattice would work!**



$L=2(l_F+l_D+2d)$



Orbit

$$x_f = -\frac{a_{xd}\theta + c_{xd}(\Delta + d\theta)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}}$$

$$x_d = -\frac{a_{xf}\theta - c_{xf}(\Delta - d\theta)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}}$$

$$a_{xf} = \cos\varphi_f;\ b_{xf} = \frac{\sin\varphi_f}{\omega_f};c_{xf} = -\omega_f \sin\varphi_f$$

$$a_{xd} = \cosh\varphi_d;\ b_{xd} = \frac{\sinh\varphi_d}{\omega_d};c_{xd} = \omega_d \sinh\varphi_d$$

$$a_{yf} = \cosh\varphi_f;\ b_{yf} = \frac{\sinh\varphi_f}{\omega_f};c_{yf} = \omega_f \sinh\varphi_f$$

$$a_{yd} = \cos\varphi_d;\ b_{yd} = \frac{\sin\varphi_d}{\omega_d};c_{yd} = -\omega_d \sinh\varphi_d$$

$$\omega_d = \sqrt{-\frac{eG_d}{pc}};\omega_f = \sqrt{\frac{eG_f}{pc}}.\qquad \varphi_{f,d} = \omega_{f,d} l_{f,d}$$

Stability: both horizontal and vertical motion must be stable

$$M_t = M_d D M_f = \begin{bmatrix} a_t & b_t \\ c_t & d_t \end{bmatrix}; \tilde{M}_t = M_f D M_d = \begin{bmatrix} d_t & b_t \\ c_t & a_t \end{bmatrix}$$

$$\boxed{\begin{gathered} 1 > \cos\mu_{x,y} = 1 + 2 b_{x,yt} c_{x,yt} > -1; \Leftrightarrow 0 > b_{x,yt} c_{x,yt} > -1 \\ \beta_{x,yF} = \sqrt{-\frac{d_{x,yt} b_{x,yt}}{a_{x,yt} c_{x,yt}}}; \beta_{x,yD} = \sqrt{-\frac{a_{x,yt} b_{x,yt}}{d_{x,yt} c_{x,yt}}}. \end{gathered}}$$

- All calculations can be done analytically and exactly
- At the stability line $|G_d| l_d = G_f l_f$ when the focal strength of F and D quad are equal, the cell us stable at any energy above a cut-off energy

$$b(E_{\min}) c(E_{\min}) = -1$$

- Off the stability line, the stability is limited at both low energy and at high energy, e.g. the operational energy range for FFAG is limited by $r = E_{\max} / E_{\max}$
- The later is a function of lattice parameters
- **There are advantages to veer of the stability line: the orbit deviations and power of synchrotron radiation can be significantly reduced**
- **Our choice for FFAG II has** $\lambda = 0.25; \varepsilon = 0.161$ **and deviated from the line by** $\delta\varepsilon = 0.034$

Dimensionless Parameterization of the Cell

$$l_f = \frac{l}{2}(1+\lambda); l_d = \frac{l}{2}(1-\lambda); \quad \varepsilon = \frac{\varphi_f - \varphi_d}{\varphi_f + \varphi_d};$$

$$\varphi = \frac{\varphi_f + \varphi_d}{2}; \varphi_f = \varphi(1+\varepsilon); \quad \varphi_d = \varphi(1-\varepsilon);$$

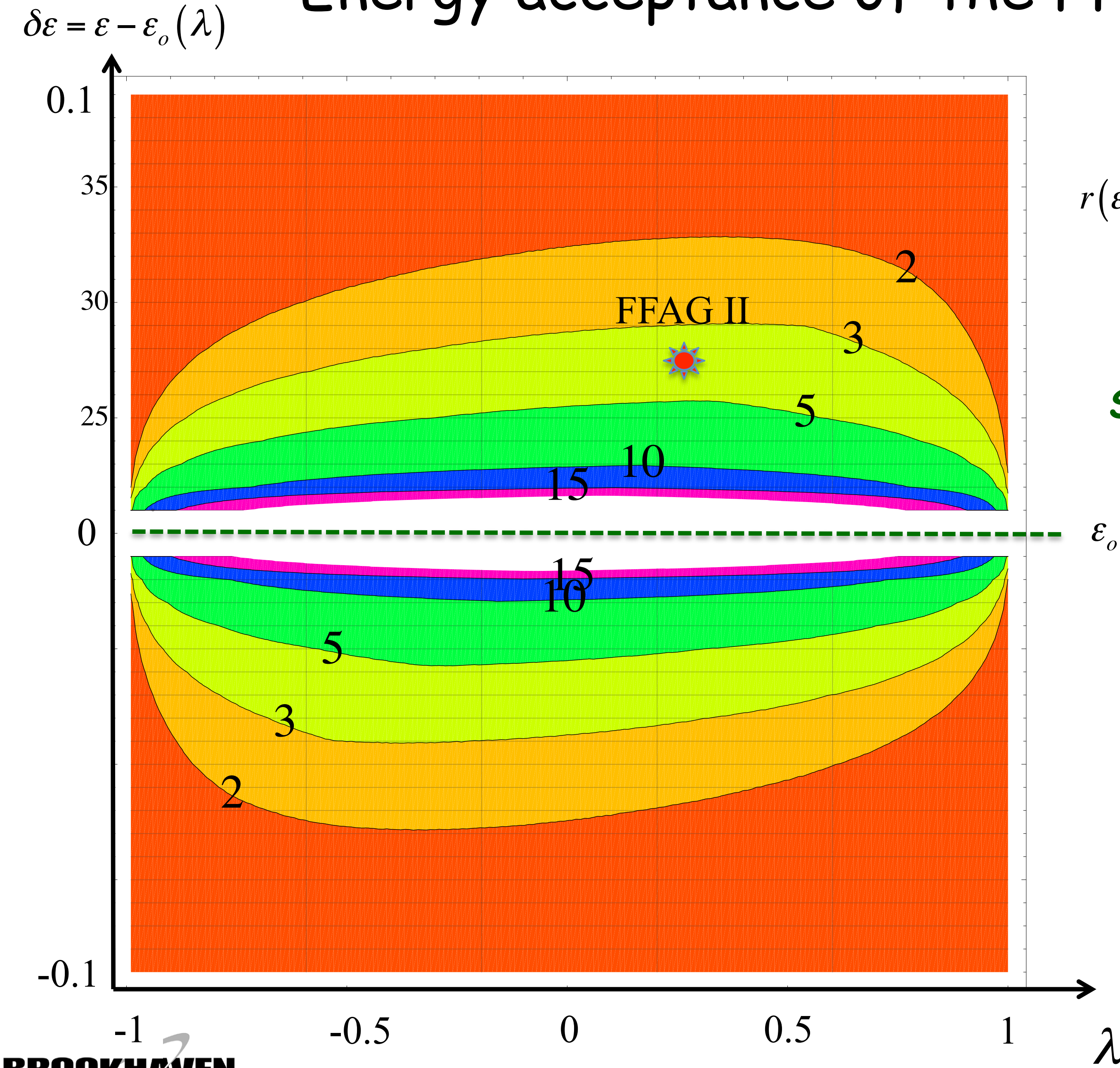
$$\omega_f = \frac{\varphi_f}{l_f} = \frac{\varphi(1+\varepsilon)}{\frac{l}{2}(1+\lambda)}; \quad \omega_d = \frac{\varphi_d}{l_d} = \frac{\varphi(1-\varepsilon)}{\frac{l}{2}(1-\lambda)};$$

Stability line

$$\lambda_o = \frac{2\varepsilon}{1+\varepsilon^2} \qquad \varepsilon = \varepsilon_o(\lambda) = \frac{1-\sqrt{1-\lambda^2}}{\lambda}$$

$$|G_d| l_d = G_f l_f$$

Energy acceptance of the FFAG



$$r(\varepsilon,\lambda) = \left(\frac{\varphi_{\max}(\varepsilon,\lambda)}{\varphi_{\min}(\varepsilon,\lambda)}\right)^2 = \frac{E_{\max}}{E_{\min}}$$

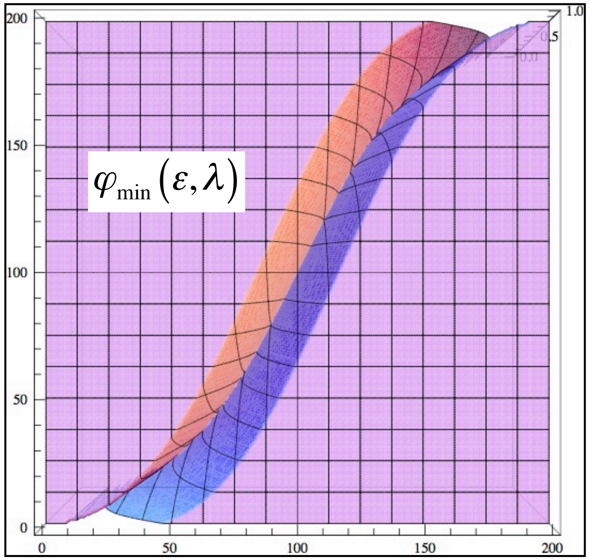
Stability line

$$\varepsilon_o(\lambda) = \frac{1-\sqrt{1-\lambda^2}}{\lambda}$$

$$r(\varepsilon,\lambda) \Rightarrow \infty$$

Minimum

$$M_t = M_d D M_f = \begin{bmatrix} a_t & b_t \\ c_t & d_t \end{bmatrix}; \tilde{M}_t = M_f D M_d = \begin{bmatrix} d_t & b_t \\ c_t & a_t \end{bmatrix}$$



$$\boxed{\begin{array}{c} 1 > \cos\mu_{x,y} = 1 + 2 b_{x,yt} c_{x,yt} > -1; \Leftrightarrow 0 > b_{x,yt} c_{x,yt} > -1 \\ \beta_{x,yF} = \sqrt{-\dfrac{d_{x,yt} b_{x,yt}}{a_{x,yt} c_{x,yt}}}; \beta_{x,yD} = \sqrt{-\dfrac{a_{x,yt} b_{x,yt}}{d_{x,yt} c_{x,yt}}}. \end{array}}$$

Dimensionless Parameterization of the Cell

$$l_f = \frac{l}{2}(1+\lambda); l_d = \frac{l}{2}(1-\lambda);$$

$$\varphi = \frac{\varphi_f + \varphi_d}{2}; \varepsilon = \frac{\varphi_f - \varphi_d}{\varphi_f + \varphi_d};$$

$$\varphi_f = \varphi(1+\varepsilon); \quad \varphi_d = \varphi(1-\varepsilon);$$

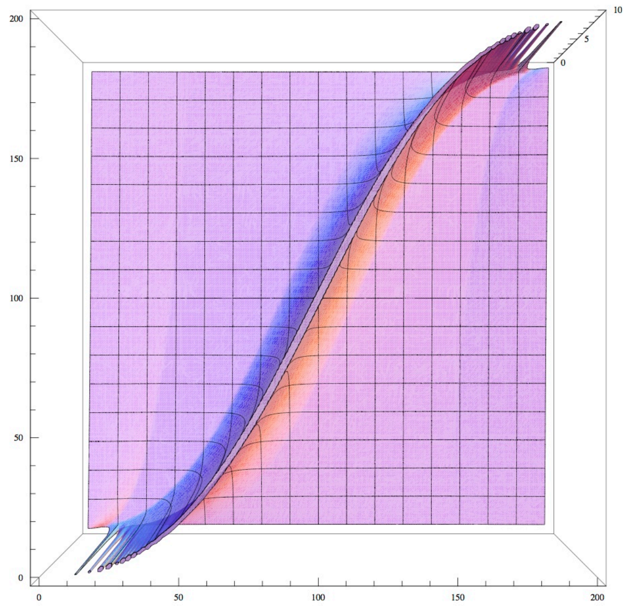
$$\omega_f = \frac{\varphi_f}{l_f} = \frac{\varphi(1+\varepsilon)}{\frac{l}{2}(1+\lambda)}; \quad \omega_d = \frac{\varphi_d}{l_d} = \frac{\varphi(1-\varepsilon)}{\frac{l}{2}(1-\lambda)};$$

$$G_f = G\left(\frac{1+\varepsilon}{1+\lambda}\right)^2; G_d = -G\left(\frac{1-\varepsilon}{1-\lambda}\right)^2;$$

Stability line

$$|G_d| l_d = G_f l_f \quad \lambda_o = \frac{2\varepsilon}{1+\varepsilon^2} \quad \varepsilon = \varepsilon_o(\lambda) = \frac{1-\sqrt{1-\lambda^2}}{\lambda}$$

Energy acceptance range



Top view 3D-plot for $r(\varepsilon,\lambda)$: $\lambda = 0.01 \cdot x - 1$ is the horizontal axis and $\varepsilon = 0.01 \cdot y - 1$ is the vertical axis. The drops at the end points are superficial because I used limited grid and some points fall of the “fault line”. Clipping is done at $r=10$.

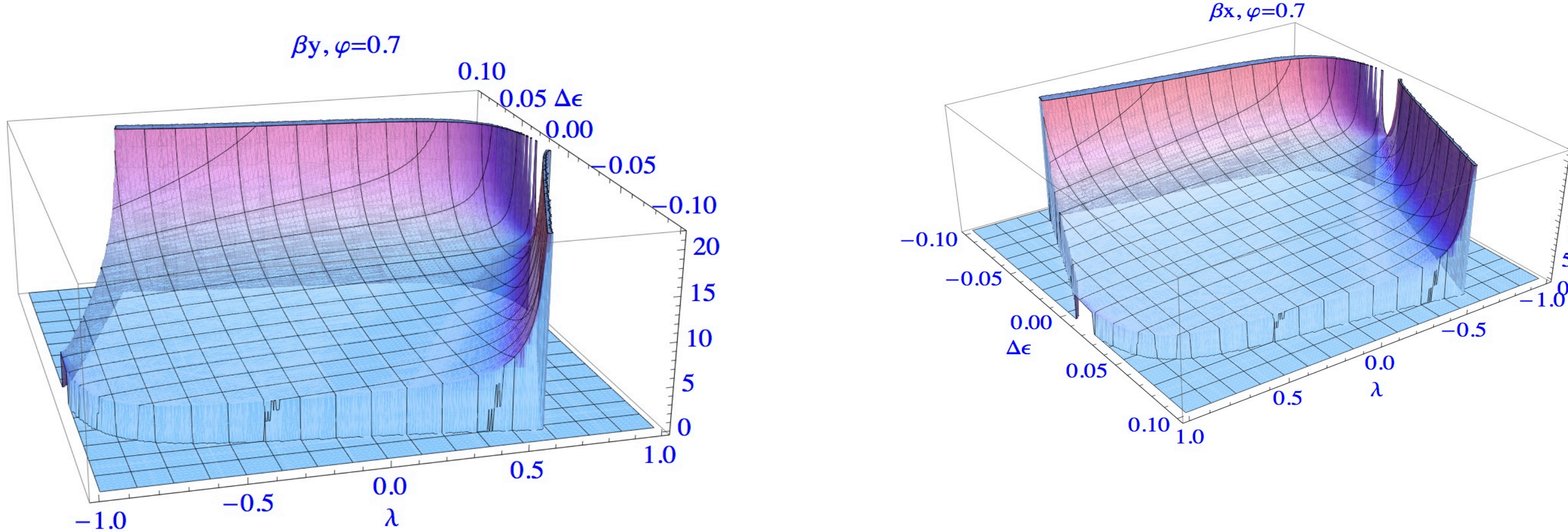


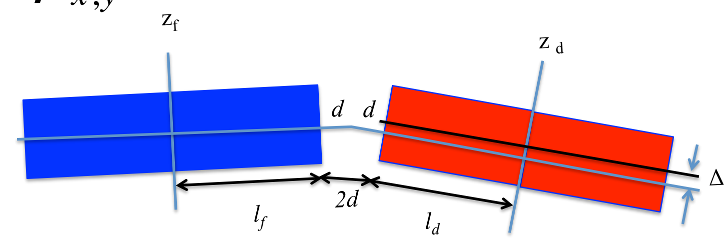
Fig. 14. Values of β_{xf}, β_{yd} in the centers of quads (i.e. maxima for the cell) around the stability line for $\varphi = 0.7$. One can see that it turns to zero where beam is unstable.

Important scaling

- Scale quadruple's gradients as $G_f \to G_f / c; G_d \to G_d / c;$
- All lengths and displacements as $l_{f,d} \to \sqrt{c} l_{f,d}; d \to \sqrt{c} d; \Delta \to \sqrt{c} \Delta$
- Then the phases in quads and the phase advances per cell stay unchanged

$$\varphi_{f,d} \to \varphi_{f,d};\ TrM_{x,y} \to TrM_{x,y};\ \beta_{x,y} \to \sqrt{c}\beta_{x,y}$$

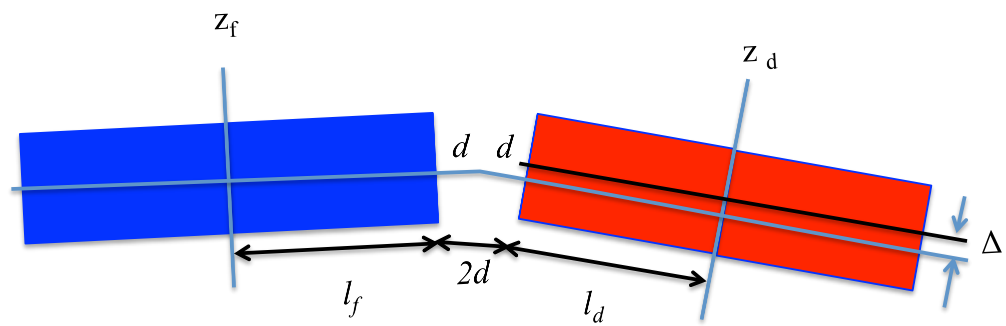
- Keep average bending radius $\theta \to \sqrt{c}\theta$



Orbits in the quads centers

$$x_f = -\frac{a_{xd}\theta + c_{xd}(\Delta + d\theta)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}} \to c \cdot x_f$$

$$x_d = -\frac{a_{xf}\theta - c_{xf}(\Delta - d\theta)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}} \to c \cdot x_d$$



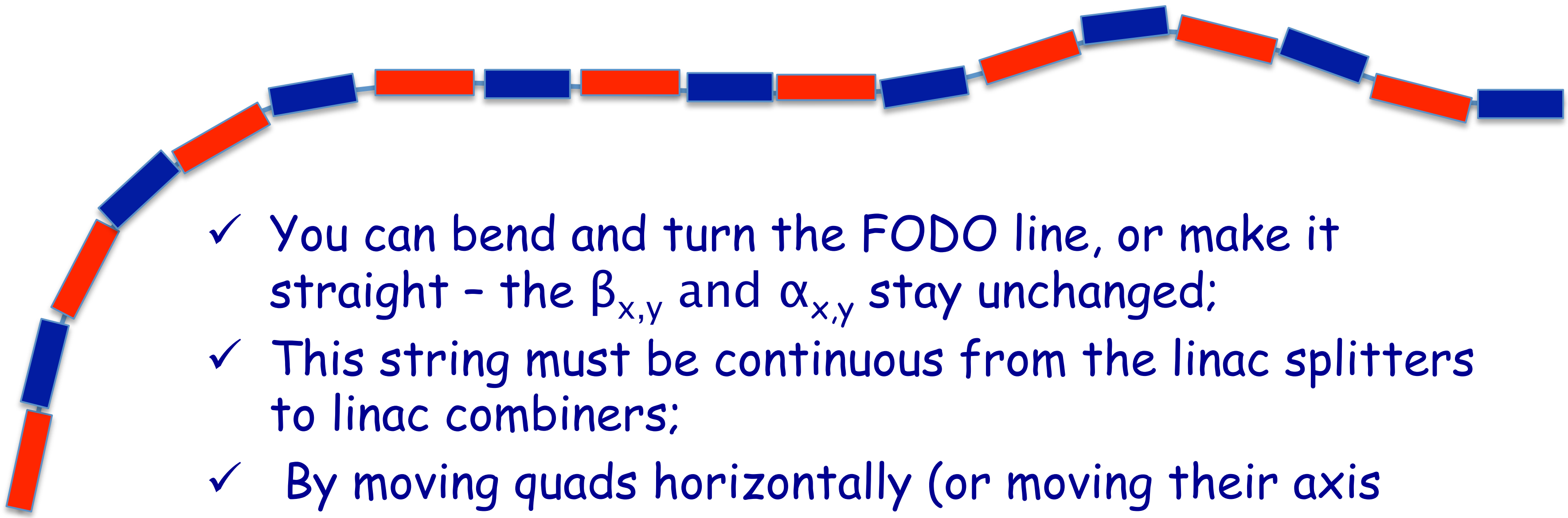
- It means that the fields on the orbits remains unchanged and, hence, the power of synchrotron radiation does not change

$$a_{xf} = \cos\varphi_f \to a_{xf};\ b_{xf} = \frac{\sin\varphi_f}{\omega_f} \to \sqrt{c} b_{xf}; c_{xf} = -\omega_f \sin\varphi_f \to c_{xf} / \sqrt{c}$$

$$a_{xd} = \cosh\varphi_d \to a_{xd};\ b_{xd} = \frac{\sinh\varphi_d}{\omega_d} \to \sqrt{c} b_{xd}; c_{xd} = \omega_d \sinh\varphi_d \to c_{xd} / \sqrt{c}$$

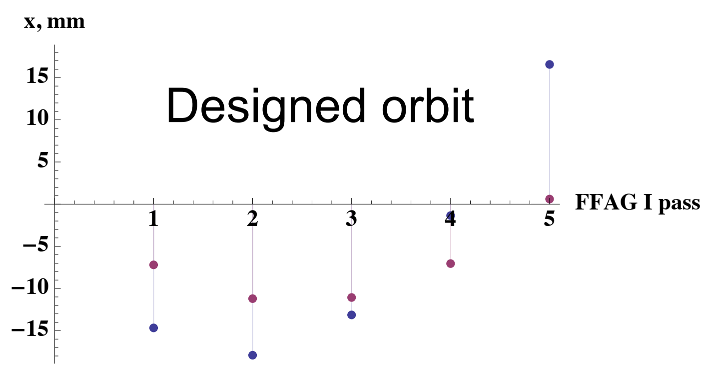
$$\boxed{B_f = G_f x_f \to B_f; B_d = G_d x_d \to B_d;\ P_{SR} \to inv}$$

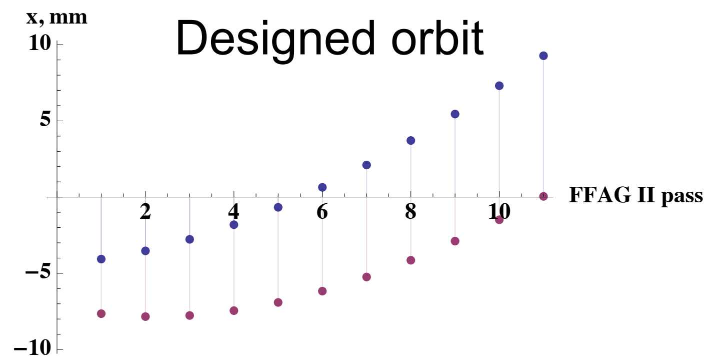
Important features



- ✓ You can bend and turn the FODO line, or make it straight – the $\beta_{x,y}$ and $\alpha_{x,y}$ stay unchanged;
- ✓ This string must be continuous from the linac splitters to linac combiners;
- ✓ By moving quads horizontally (or moving their axis using dipole trims) one can adjust all orbits to a desirable pattern, including separating a single energy beam from the rest of the pack, bending beam around detector or setting them on a single straight orbit.

Regular orbits in the arcs: quad's centers





$$x_f = -\frac{a_{xd}\theta + c_{xd}(\Delta + d\theta/2)}{a_{xf}c_{xd} + a_{xd}c_{xf} + dc_{xd}c_{xf}}$$

$$x_d = -\frac{a_{xf}\theta - c_{xf}(\Delta - d\theta/2)}{a_{xf}c_{xd} + a_{xd}c_{xf} + dc_{xd}c_{xf}}$$

$$F_i = \begin{bmatrix} \cos(\omega_i s) & \dfrac{\sin(\omega_i s)}{\omega_i} \\ -\omega_i \sin(\omega_i s) & \cos(\omega_i s) \end{bmatrix}; \quad \omega_i = \sqrt{\frac{e|G_f|}{p_i c}} = \sqrt{\frac{|G_f|}{B\rho_i}}; \quad O = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix};$$

$$D_i = \begin{bmatrix} \cosh(\Omega_i s) & \dfrac{\sinh(\Omega_i s)}{\Omega_i} \\ \Omega_i \sinh(\Omega_i s) & \cosh(\Omega_i s) \end{bmatrix}; \quad \Omega_i = \sqrt{\frac{e|G_d|}{p_i c}} = \sqrt{\frac{|G_d|}{B\rho_i}};$$

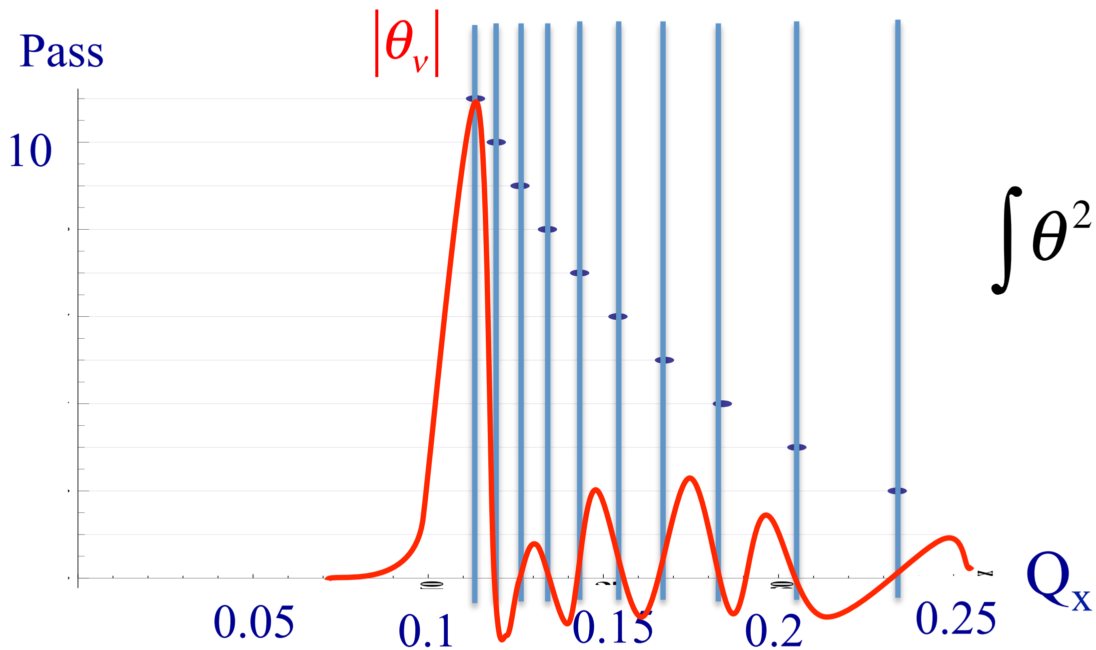
$$a_{xf} = \cos\varphi_i/2;\; b_{xf} = \frac{\sin\varphi_i/2}{\omega_i}; c_{xf} = -\omega_i \sin\varphi_i/2$$

$$a_{xd} = \cosh\phi_i/2;\; b_{xd} = \frac{\sinh\phi_i/2}{\Omega_i}; c_{xd} = \Omega_i \sinh\phi_i/2$$

General considerations

- Separating individual trajectory, especially the top energy is the most difficult, especially with 11 passes
  - First, the distance to the 10^{th} orbit is minuscular
  - The tune difference with 10^{th} orbit is minuscular
  - And "do not disturb" all 10 oscillators
- But doable, with many cells: need to zero all harmonics but one
- Since orbits are "resonantly excited" merging all trajectories into one is the easiest task

$Q_{x10}-Q_{x11}=0.119432-0.11351=0.0059$!!!



$$\int \theta^2(s)ds = \int |\theta_v|^2 d\nu$$

General considerations

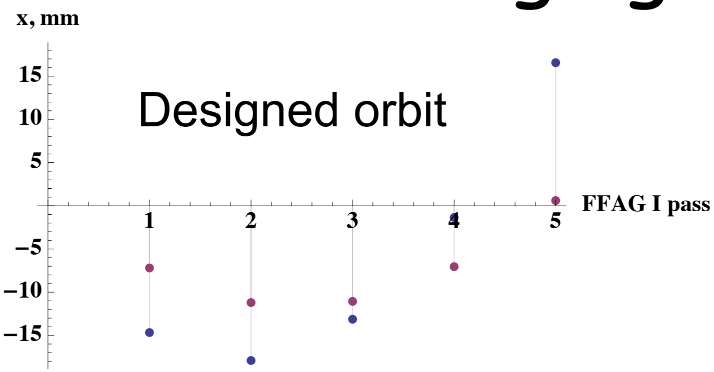
$Q_{x\ FFAGI}$={0.293262, 0.137903, 0.0937012, 0.0720672, 0.0591474}

$Q_{x\ FFAGII}$= {0.275592, 0.233907, 0.205245, 0.183984, 0.167454, 0.154172, 0.143232, 0.134045, 0.126207, 0.119432, 0.11351}

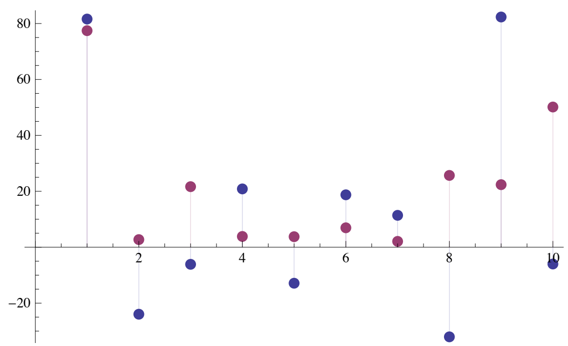
$Q_{x10}-Q_{x11}=0.119432-0.11351=0.0059$!!!

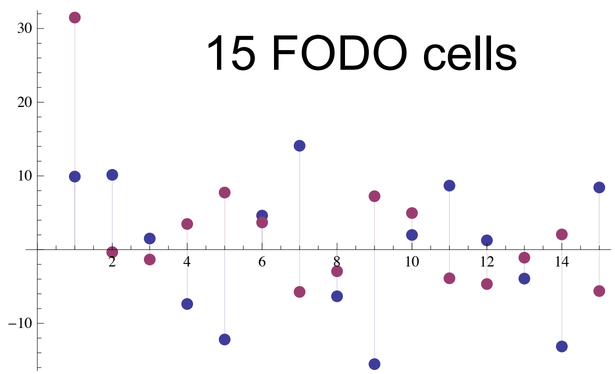
- Uncertainty principle $N_{cells}\Delta\nu \sim 1/\pi$
- To make an efficient "separator" we will need to use multiple FODO cells (~ 60 in case of FFAG II)
- Using "just sufficient" number cells (22 in case of FFAG II) would lead to HUGE kicks
- Merging requires fewer cells than the separating a single orbit

Merging orbits in FFAG I

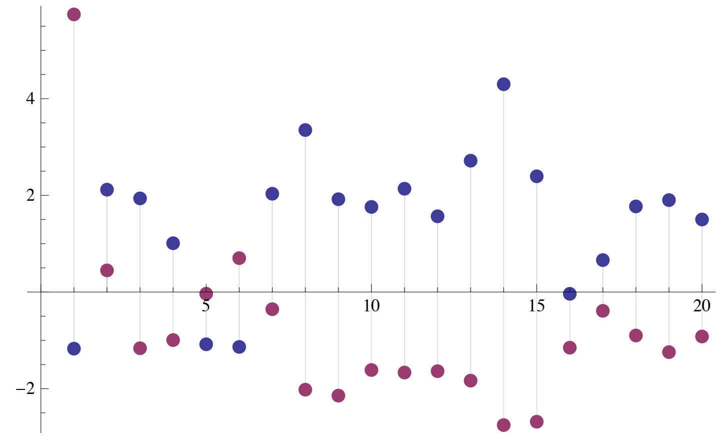


5 orbits ->Min 10 FODO cells
Requires huge (80 mm) shifts

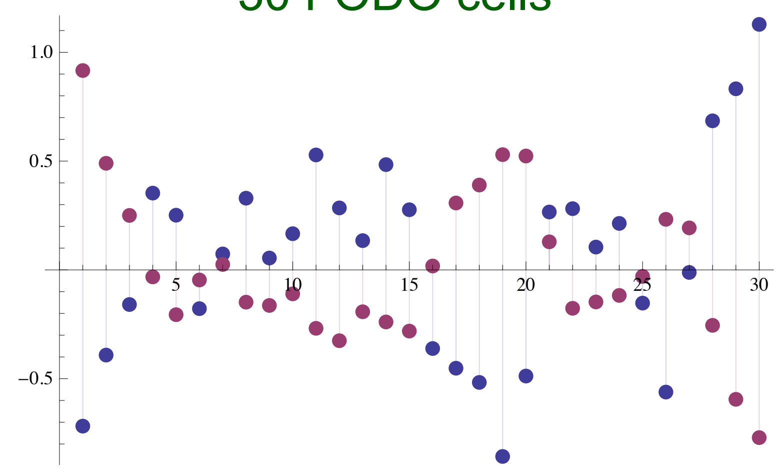




20 FODO cells



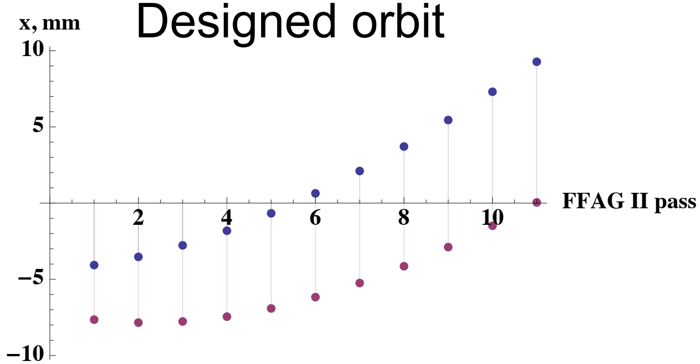
30 FODO cells



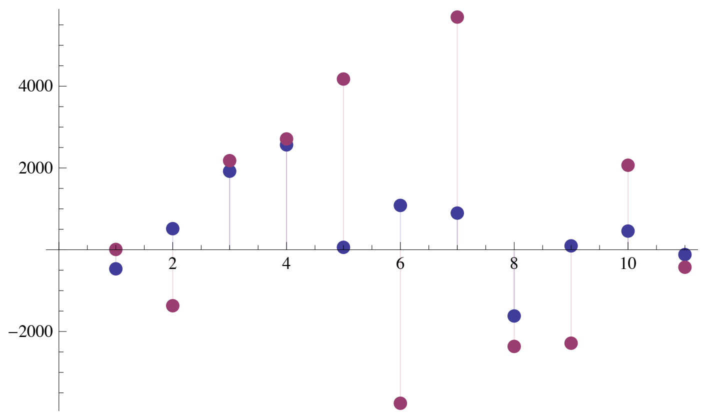
{x,x'}={1.62E-12, 2.22E-15, 5.54E-13, 1.26E-13, -8.56E-13, -6.48E-14, 6.06E-14, 7.78E-13, -1.68E-13,-1.49E-13}

Merging orbits in FFAG II

Designed orbit

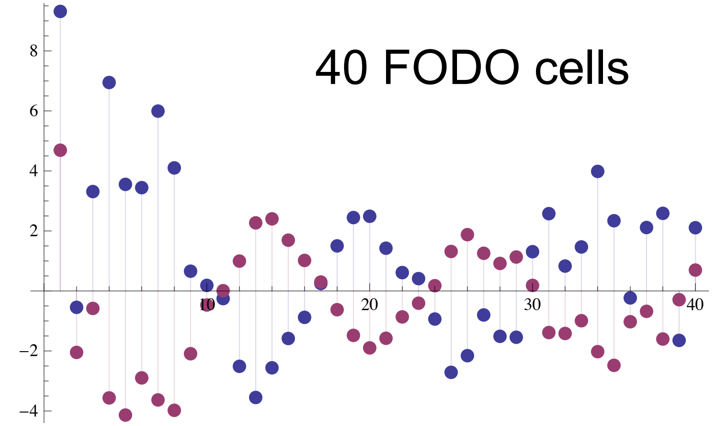


11 orbits -> Min 11 FODO cells
Requires huge (5000+ mm) displacements – 250 T kicks – just a junk!

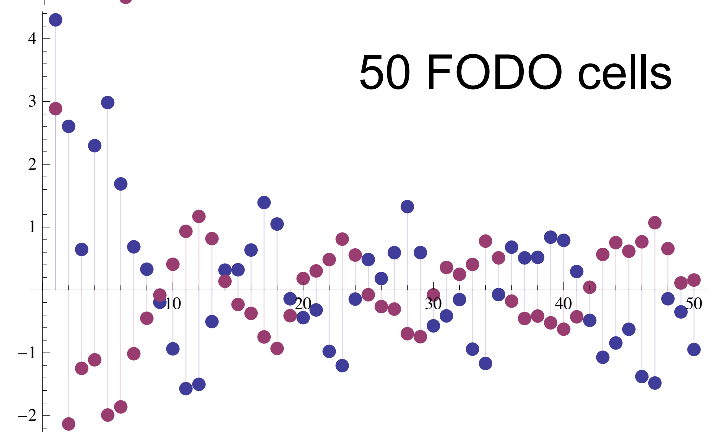


Error{x,x'}<3E-12

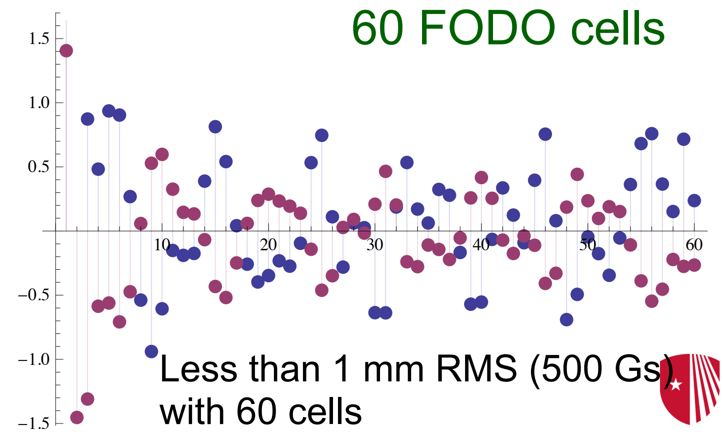
40 FODO cells



50 FODO cells

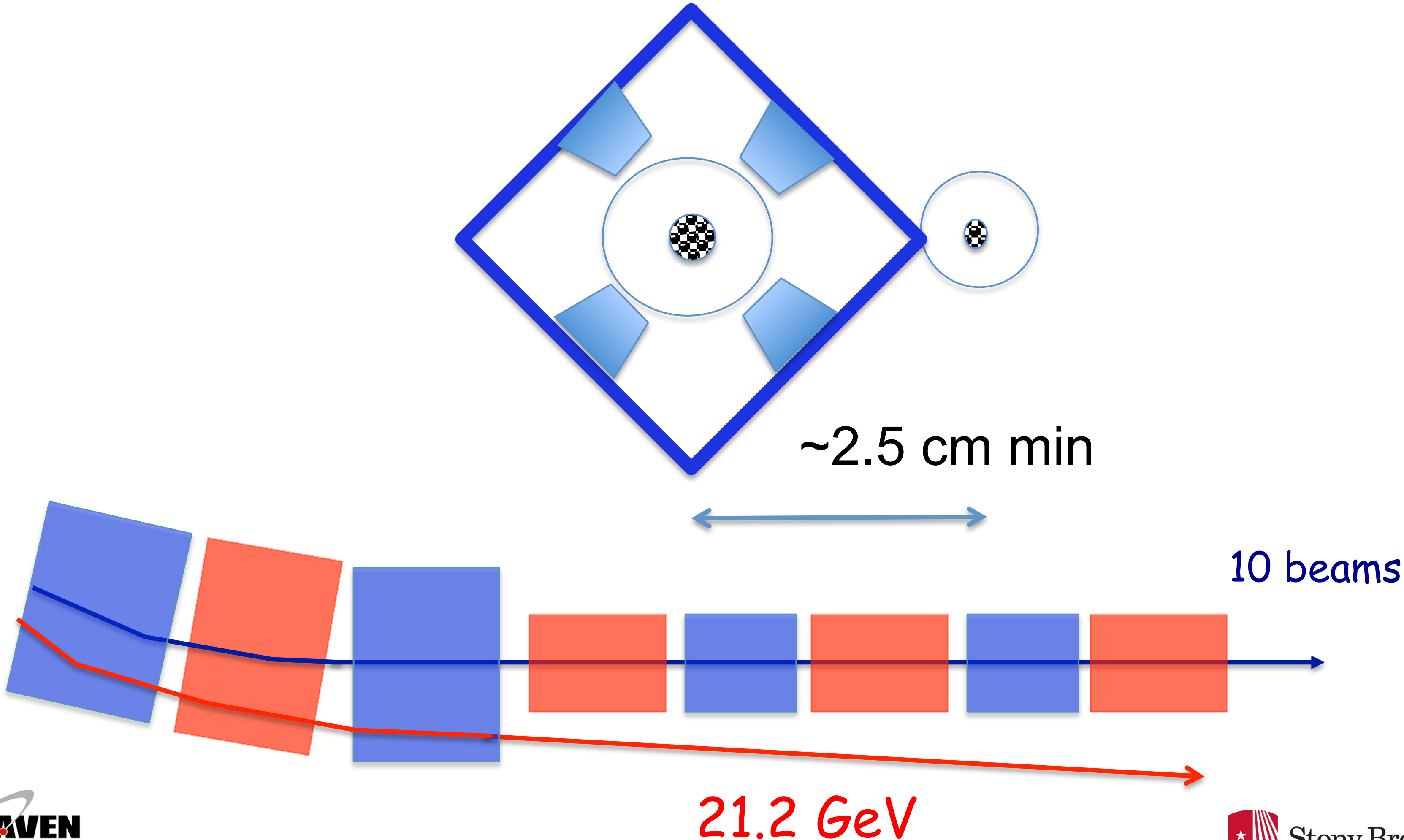


60 FODO cells



Less than 1 mm RMS (500 Gs) with 60 cells

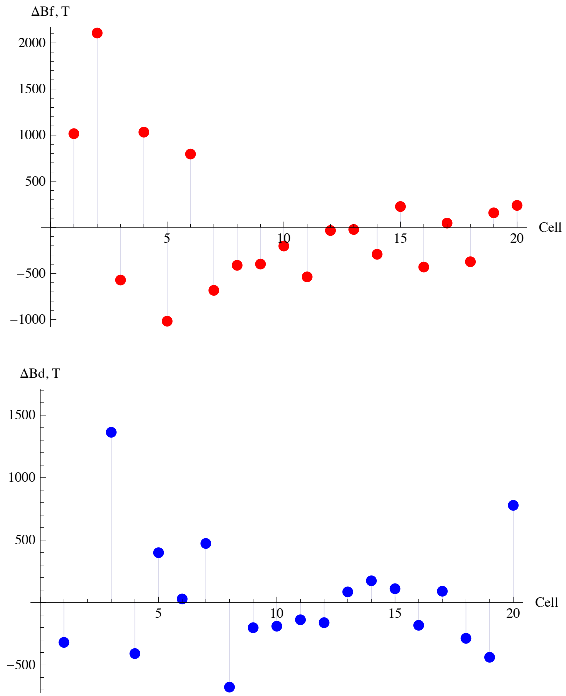
Separating a single energy beam from the rest: toughest



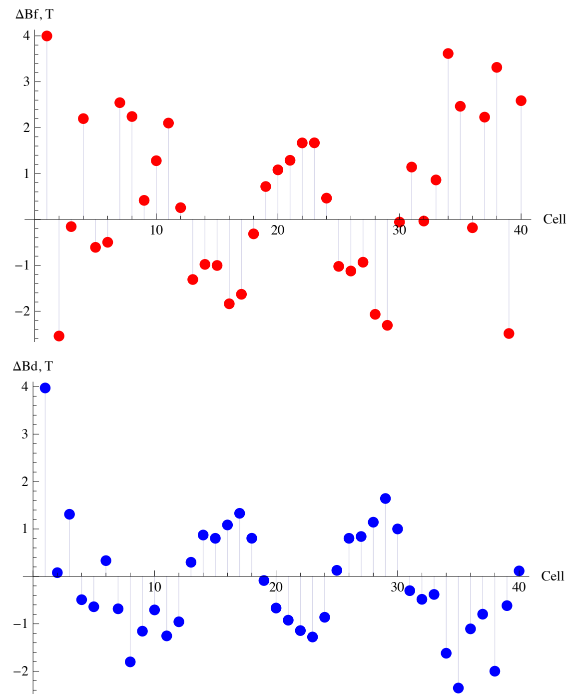
Separating single bunch in FFAG II by 25 mm, 6 mrad with G=12.5 T/m

CASE Center for Accelerator Science and Education

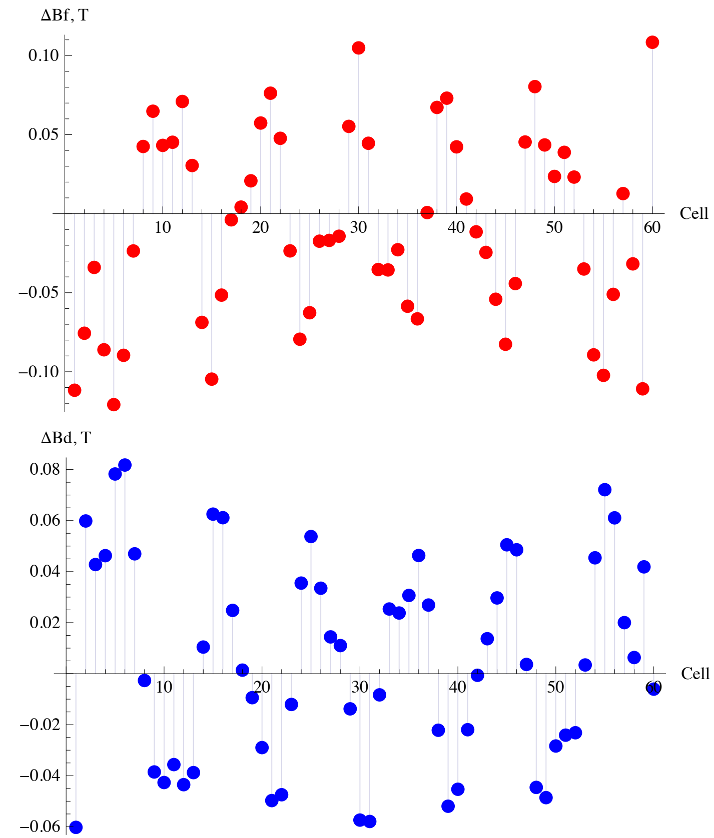
20 cells: 2KT



40 cells: 4T



60 cells: 0.1T



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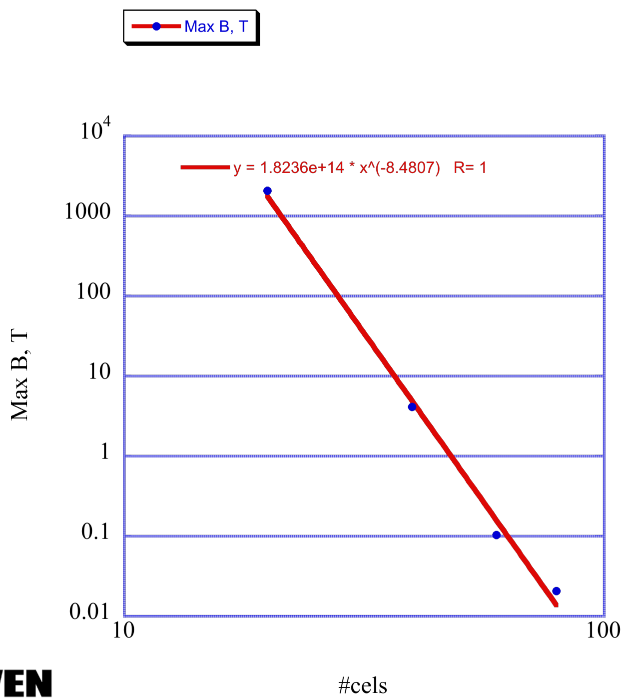
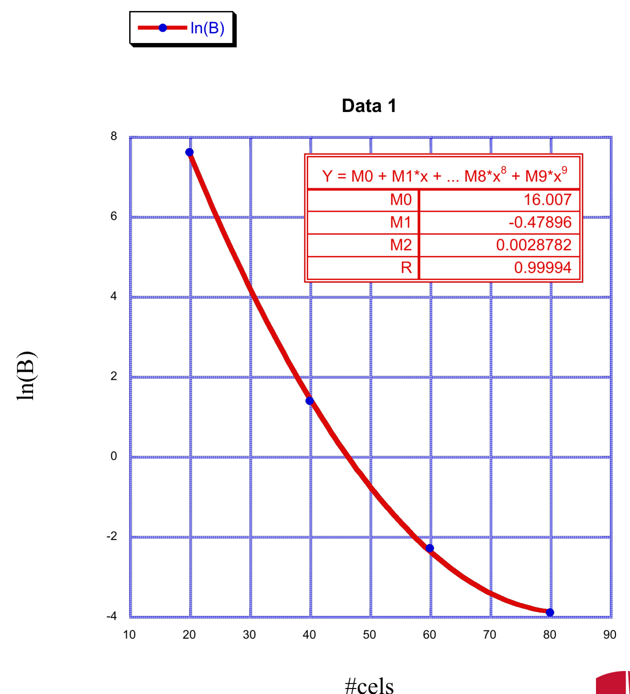
Scaling

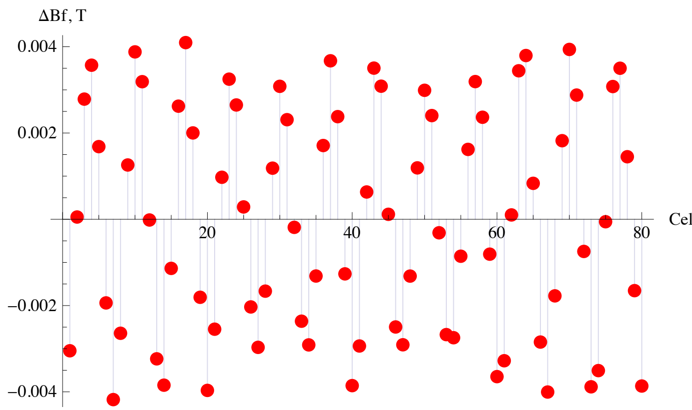
- Value of the required kicks is a high (nearly exponential!) funnction of 1/N, where N is the number of used cells. It is quite natural since we want to kepp all but one oscillators quiet, while exciting one of them. This is usually calls for an adiabatic excitation, which is naturally (Poincare) associated with exponential dependence on the transient time. Naturally at very large cell number one should expect ~1/Ncell dependence, simply from the required total kick

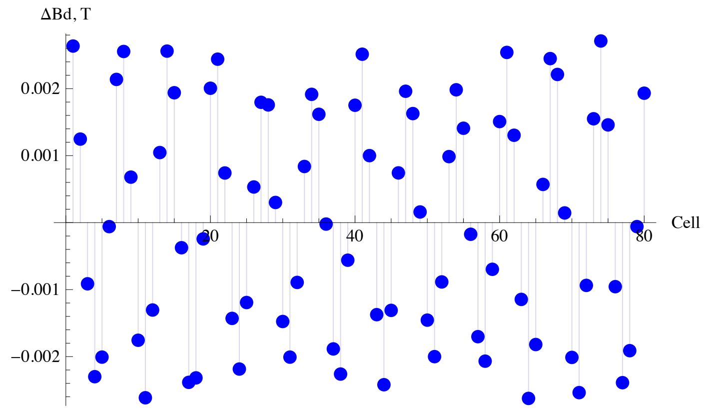


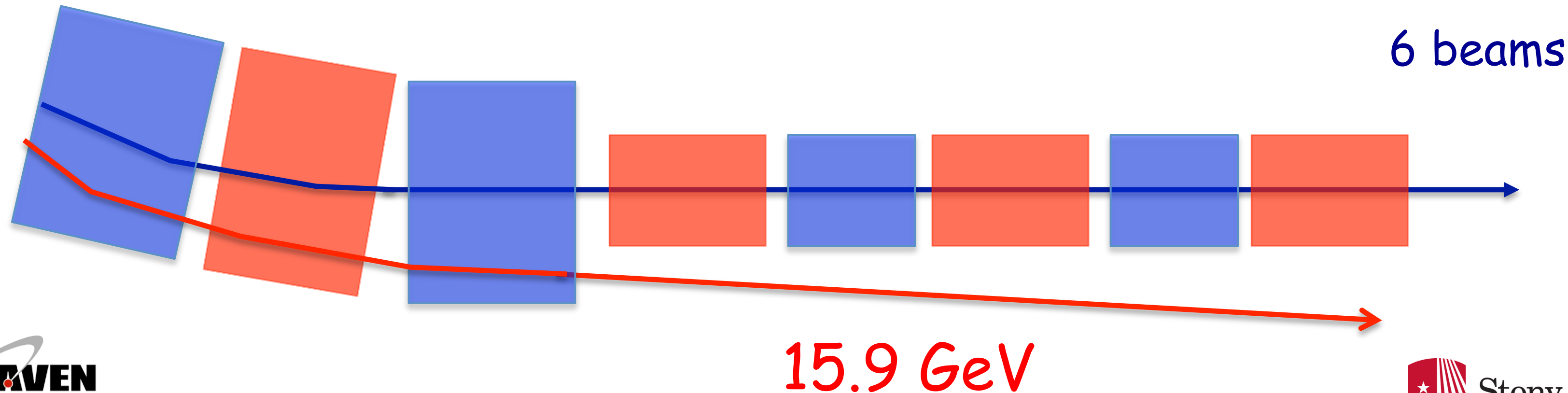


Separating single bunch in FFAG II by 25 mm, 25 mm, 6 mrad with G=12.5 T/m

80 cells: 0.004T (40 Gs)

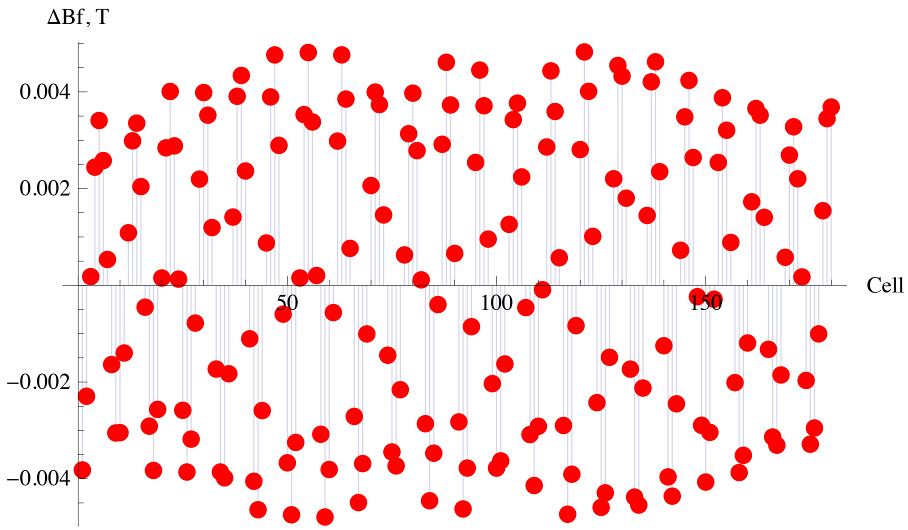


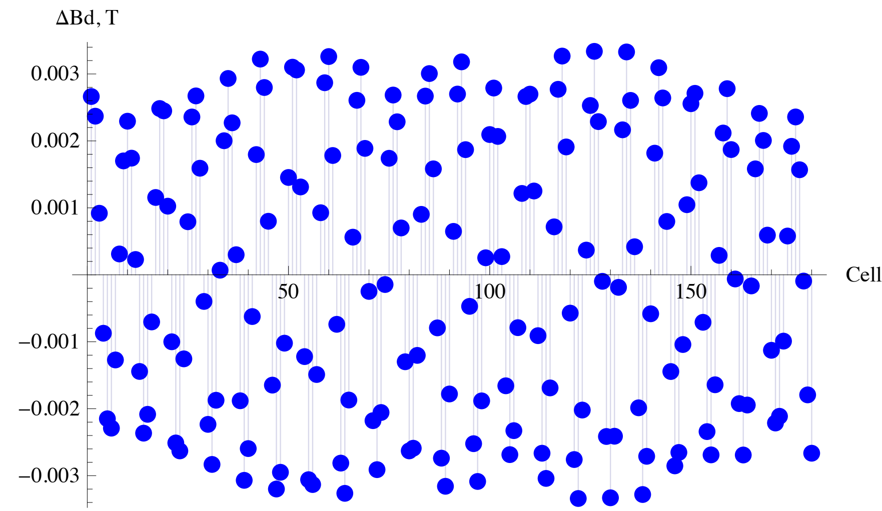


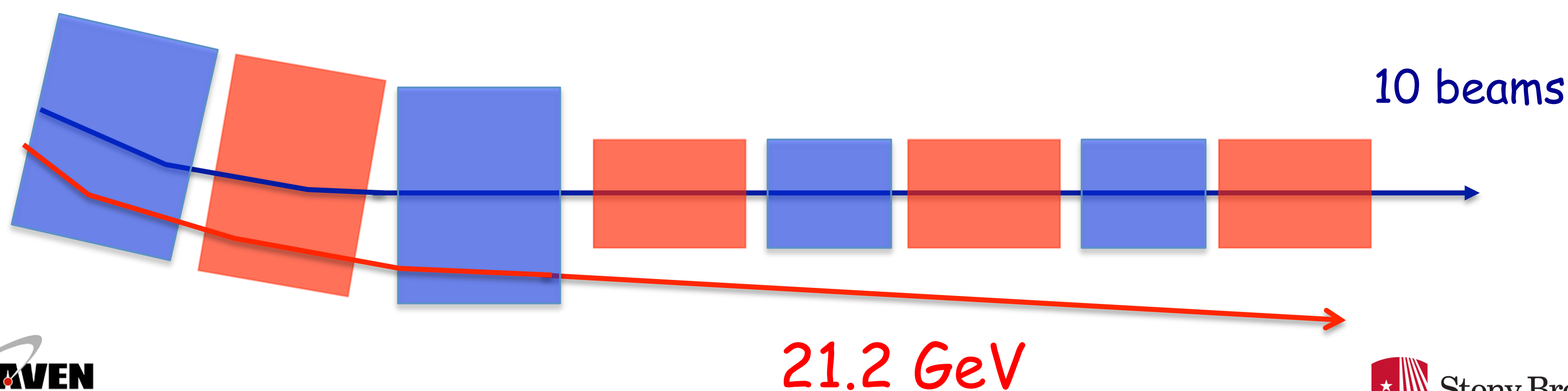


Separating single bunch in FFAG II by 25 mm, 6 mrad with G=50 T/m

180 cells: max ~ 0.005T (50 Gs)







Conclusion

- Combination of FFAG arcs with splitters and combiners is an excellent cost-effective combination for multi-pass ERL:
  - Each FFAG arcs propagate beams with multiple energies
  - Splitters and combiners allow to (a) compensate for the time-of-flight and R56 errors accumulated in FFAG; (b) match each individual beam with linac
- Using a paraxial pure-quad FFAG gives enormous flexibility
  - one can bend and turn the beams without loosing control of the optics
- Merging beams from FFAG arc into a single on-axis orbit can be done exactly without any limitations. Separating 21.2 or 15.9 (or any other energy beam is possible): it is possible top combine both separation ~ 1" plus 5 to 8 mrad angle;
- Separation can be easily combined with merging the remaining beams onto a single axis of the straight section. A combiner just a "flip" of a separator
- Using low gradient beamline make separation practical – it can be done with trims within 100 Gs range.
- Using 50 T/m quads in eRHIC makes separation ~ 25 mm tough…
  - Another option for 50 T/m quads is to keep the top energy separated from the rest of the beam for entire pass (third arc) after the linac splitter.

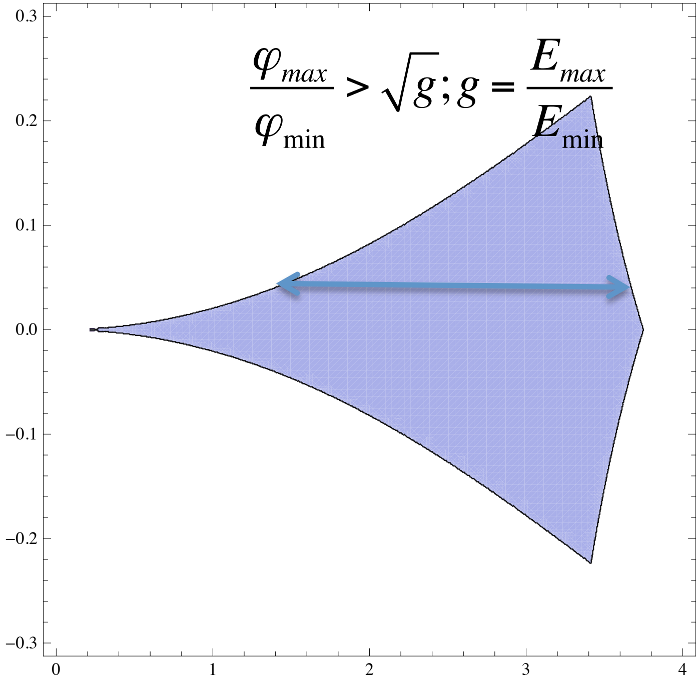
……BACK-UPS………

Important scaling

- Let's increase quads aperture as c
- Scaling does no change peak filed on the quad's pole-tips
- Aperture increases
- Power of synchrotron radiation does not changes
- Orbits are separated further (which convenient for splitting the colliding beam)
- Tolerances to the quads manufacturing and position are reducing
- Number of the elements, controls, BPMs, diagnostics, vacuum chambers is reducing
- Worth considering as a value engineering exercise….

All distances in units of Lθ

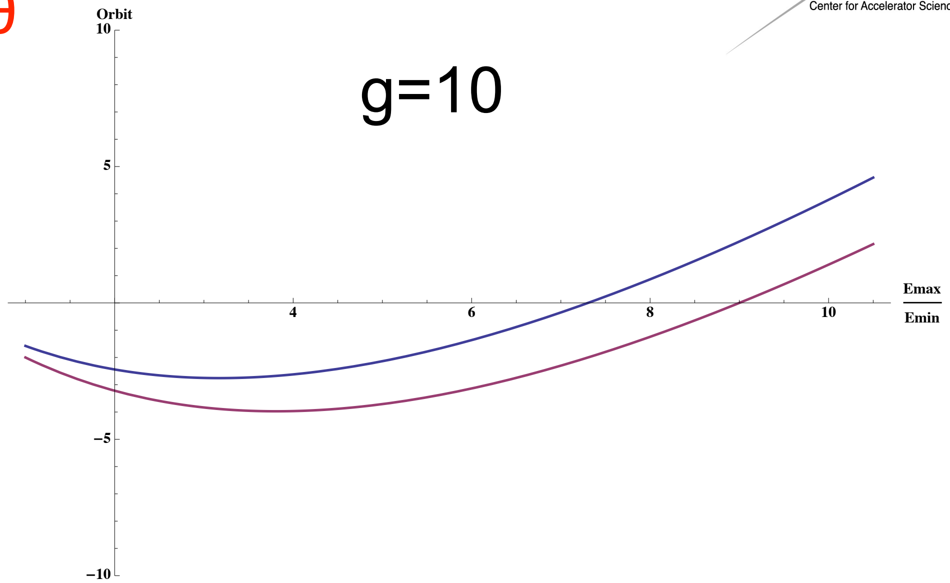
Length asymmetry



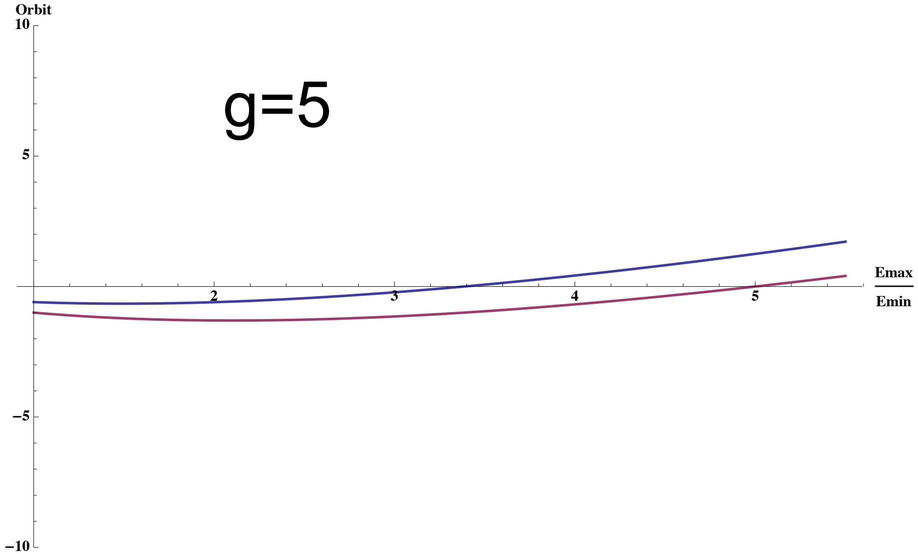
$$\frac{\varphi_{max}}{\varphi_{\min}} > \sqrt{g}; g = \frac{E_{max}}{E_{\min}}$$

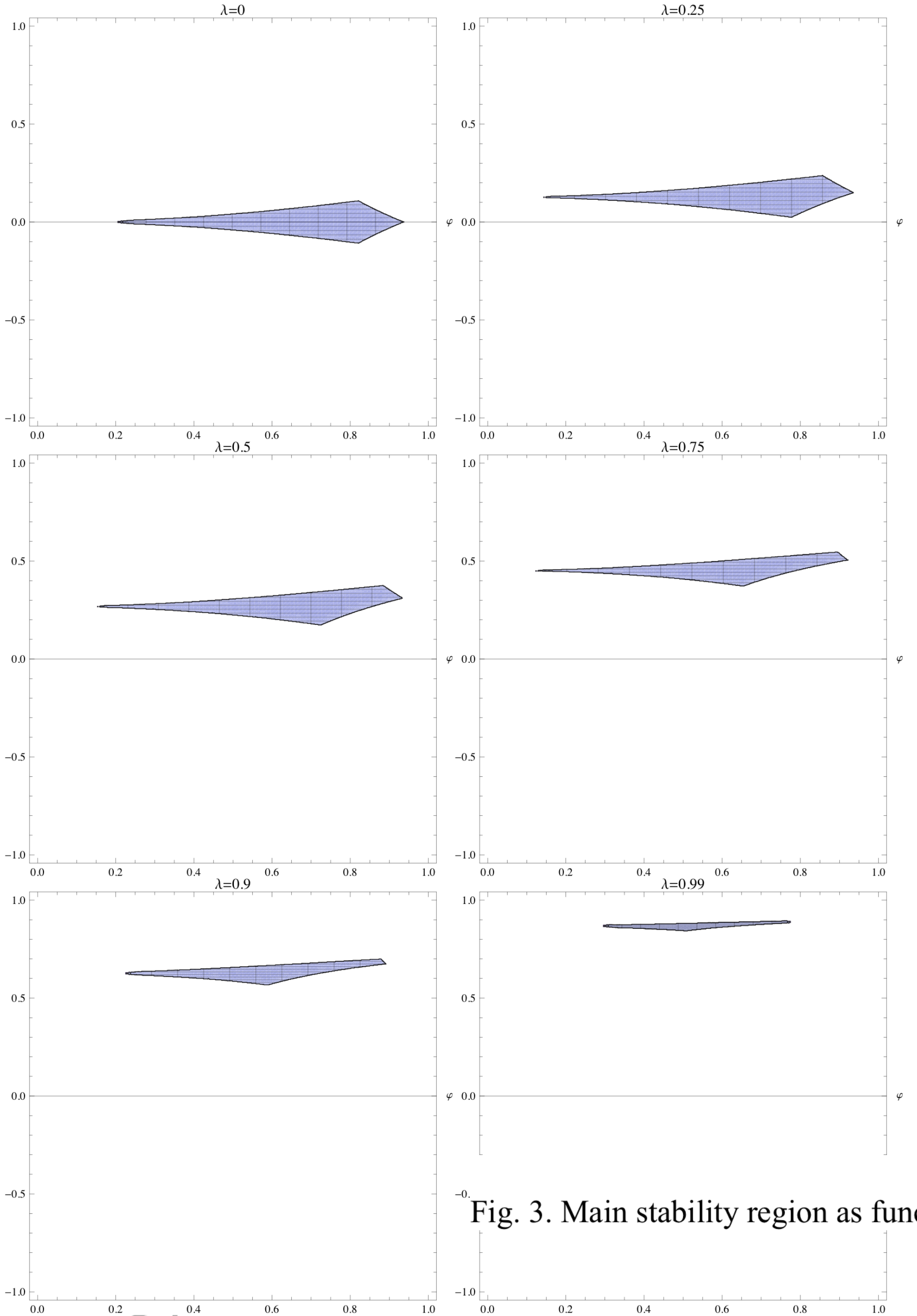
Phase

g=10



g=5





$$1 > \cos\mu_{x,y} = 1 + 2b_{x,yt}c_{x,yt} > -1; \Leftrightarrow 0 > b_{x,yt}c_{x,yt} > -1$$

$$\beta_{x,yF} = \sqrt{-\frac{d_{x,yt}b_{x,yt}}{a_{x,yt}c_{x,yt}}}; \beta_{x,yD} = \sqrt{-\frac{a_{x,yt}b_{x,yt}}{d_{x,yt}c_{x,yt}}}.$$

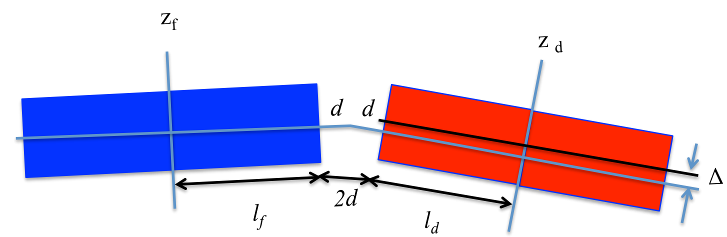
Fig. 3. Main stability region as function of λ. Vertical axis is ε.

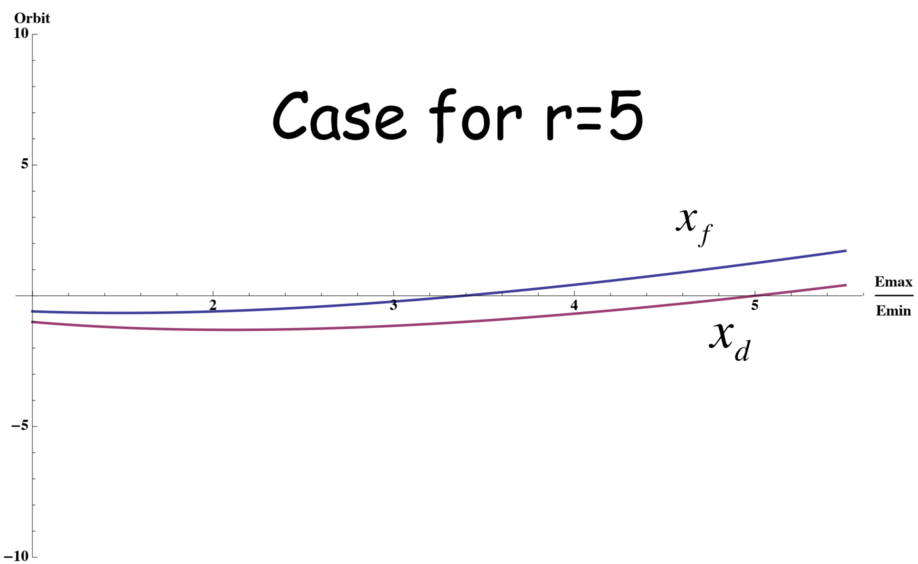
Optimizing the the SR power

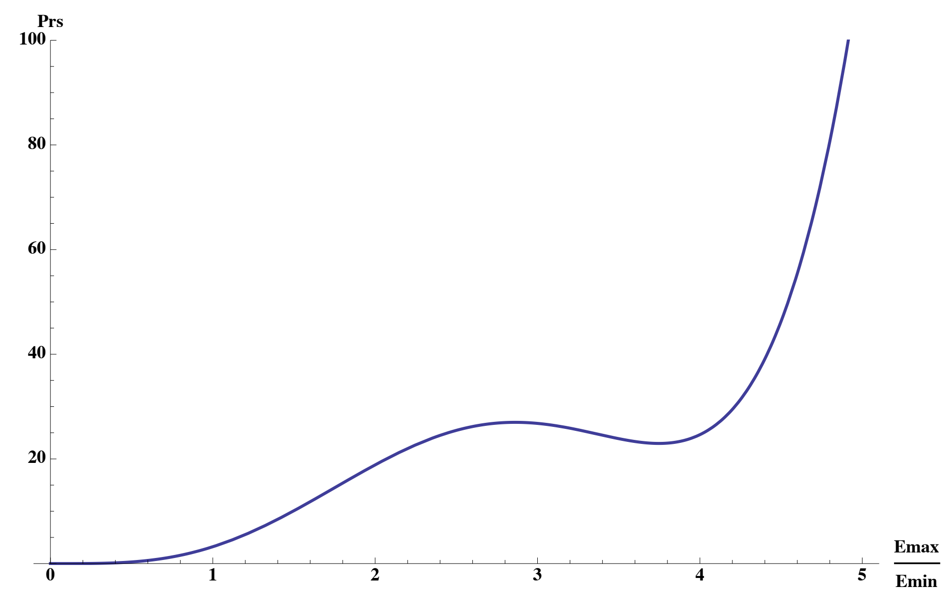
Optimization includes the cells structure &
It is very important to optimized shift between the magnetic axes of F and D quads Δ

$$x_f = -\frac{a_{xd}\theta + c_{xd}\left(\Delta + d\theta\right)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}}$$

$$x_d = -\frac{a_{xf}\theta - c_{xf}\left(\Delta - d\theta\right)}{a_{xf}c_{xd} + a_{xd}c_{xf} + 2dc_{xd}c_{xf}}$$







Separating single bunch in FFAG II by 25 mm, 25 mm, 6 mrad with G=50 T/m

80 cells: ~0.1T (1 kGs)